

Powers Chart

Powers Chart - **Base**^{Power}

	Power	2	3	4	5	6	7	8	9	10
Base										
2		4	8	16	32	64	128	256	512	1024
3		9	27	81	243	729				
4		16	64	256	1024					
5		25	125	625						
6		36	216	1296						
7		49	343							
8		64	512							
9		81	729							
10		100	1000							
11		121								
12		144								
13		169								
14		196								
15		225								

$$\begin{array}{l}
 2^2 = 4 \\
 2^1 = 2 \\
 2^0 = 1 \\
 2^{-1} = \frac{1}{2^1} = \frac{1}{2} \\
 2^{-2} = \frac{1}{2^2} = \frac{1}{4} \\
 2^{-3} = \frac{1}{8} \\
 2^{-4} = \frac{1}{16} \\
 2^{-5} = \frac{1}{32} \\
 2^{-6} = \frac{1}{64} \\
 2^{-7} = \frac{1}{128}
 \end{array}
 \quad
 \begin{array}{l}
 3^2 = 9 \\
 3^1 = 3 \\
 3^0 = 1 \\
 3^{-1} = \frac{1}{3} \\
 3^{-2} = \frac{1}{3^2} = \frac{1}{9} \\
 3^{-3} = \frac{1}{27} \\
 3^{-4} = \frac{1}{81} \\
 3^{-5} = \frac{1}{243} = \frac{1}{3^5}
 \end{array}
 \quad
 \begin{array}{l}
 4^2 = 16 \\
 4^1 = 4 \\
 4^0 = 1 \\
 4^{-1} = \frac{1}{4} \\
 4^{-2} = \frac{1}{16} \\
 4^{-3} = \frac{1}{64}
 \end{array}
 \quad
 \begin{array}{l}
 5^2 = 25 \\
 5^1 = 5 \\
 5^0 = 1 \\
 5^{-1} = \frac{1}{5} \\
 5^{-2} = \frac{1}{25} \\
 5^{-3} = \frac{1}{125}
 \end{array}
 \quad
 \begin{array}{l}
 6^2 = 36 \\
 6^1 = 6 \\
 6^0 = 1 \\
 6^{-1} = \frac{1}{6} \\
 6^{-2} = \frac{1}{36} \\
 6^{-3} = \frac{1}{216}
 \end{array}$$

Properties of Rational Exponents

Property

1. $a^m \cdot a^n = a^{m+n}$

$$12^{\frac{1}{4}} \cdot 12^{\frac{5}{4}} = 12^{\frac{1}{4} + \frac{5}{4}} = 12^{\frac{3}{4} + \frac{1}{4}} = 12^{\frac{2+3}{4}} = 12^{\frac{5}{4}}$$

2. $(a^m)^n = a^{mn}$

$$(5^{\frac{1}{3}})^2 = 5^{(\frac{1}{3}) \cdot 2} = 5^{\frac{2}{3}} = \sqrt[3]{5^2}$$

3. $(ab)^m = a^m b^m$

$$(5^{\frac{1}{3}} \cdot 7^{\frac{1}{4}})^3 = (5^{\frac{1}{3}})^3 \cdot (7^{\frac{1}{4}})^3 = 5^1 \cdot 7^{\frac{3}{4}}$$

4. $a^{-m} = \frac{1}{a^m}, a \neq 0$

$$5^{-\frac{1}{2}} = \frac{1}{5^{\frac{1}{2}}} = \frac{1}{\sqrt{5}}$$

5. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$

$$\frac{10}{10^{\frac{7}{5}}} = \frac{10^1}{10^{\frac{7}{5}}} = 10^{1 - \frac{7}{5}} = 10^{\frac{5-7}{5}} = 10^{-\frac{2}{5}}$$

6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

$$\left(\frac{56^{\frac{1}{4}}}{7^{\frac{1}{4}}}\right)^5 = \frac{(56^{\frac{1}{4}})^5}{(7^{\frac{1}{4}})^5} = \frac{(\sqrt[4]{56})^5}{(\sqrt[4]{7})^5} = \left(\frac{\sqrt[4]{56}}{\sqrt[4]{7}}\right)^5 = \left(\sqrt[4]{8}\right)^5 = 8^{\frac{5}{4}}$$

PROPERTIES OF RADICALS

Product Property of Radicals

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Quotient Property of Radicals

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$$

1. $(6^6 \cdot 5^6)^{-1/6} = (6^6)^{-\frac{1}{6}} \cdot (5^6)^{-\frac{1}{6}}$
 $= 6^{-1} \cdot 5^{-1} = \frac{1}{6} \cdot \frac{1}{5} = \frac{1}{30}$

2. $\frac{\sqrt{245}}{\sqrt{5}} = \sqrt{\frac{245}{5}} = \sqrt{49} = 7$

Simplifying Rational Exponent Expressions 85

SIMPLEST FORM A radical with index n is in **simplest form** if the radicand has no perfect n th powers as factors and any denominator has been rationalized.

No perfect n^{th} powers inside the radical
No radical in denominator

$$3. \sqrt[3]{\frac{5}{9}} = \frac{\sqrt[3]{5}}{\sqrt[3]{9}} = \frac{\sqrt[3]{5}}{\sqrt[3]{3 \cdot 3}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \frac{\sqrt[3]{15}}{\sqrt[3]{27}} = \frac{\sqrt[3]{15}}{3}$$

LIKE RADICALS Radical expressions with the same index and radicand are **like radicals**. To add or subtract like radicals, use the distributive property.

$$4. 6\sqrt[4]{6} + 2\sqrt[4]{6} = 8\sqrt[4]{6}$$

$$5. \sqrt[3]{8x^7y^3z^{11}}$$

$$\sqrt[3]{\underbrace{2 \cdot 2 \cdot 2}_{\text{circled}} \cdot \underbrace{x \cdot x \cdot x}_{\text{circled}} \cdot \underbrace{x \cdot x \cdot x}_{\text{circled}} \cdot \underbrace{y \cdot y \cdot y}_{\text{circled}} \cdot \underbrace{z \cdot z \cdot z}_{\text{circled}} \cdot \underbrace{z \cdot z \cdot z}_{\text{circled}} \cdot \underbrace{z \cdot z \cdot z}_{\text{circled}} \cdot \underbrace{z \cdot z \cdot z}_{\text{circled}} \cdot \underbrace{z \cdot z \cdot z}_{\text{circled}}}$$

$$2 \cdot x \cdot x \cdot y \cdot z \cdot z \cdot z \sqrt[3]{x \cdot z \cdot z} = 2x^2y \cdot z^3 \sqrt[3]{xz^2}$$

$$6. 7\sqrt[3]{2a^5} - a\sqrt[3]{128a^2}$$

$$7\sqrt[3]{\underbrace{2a^3a^2}_{\text{circled}}} - a\sqrt[3]{\underbrace{2 \cdot 2 \cdot 2}_{\text{circled}} \cdot \underbrace{2 \cdot 2 \cdot 2}_{\text{circled}} \cdot 2 \cdot a \cdot a}$$

$$7 \cdot a \sqrt[3]{2a^2} - a \cdot 2 \cdot 2 \sqrt[3]{2a^2}$$

$$7a \sqrt[3]{2a^2} - 4a \sqrt[3]{2a^2}$$

$$3a \sqrt[3]{2a^2}$$